

# CONTENTS

## PREFACE

xvii

<b>1</b>	<b>PRELIMINARIES</b>	<b>1</b>
1.	Coordinate system in a $V_n$	1
2.	Coordinate transformation laws of various geometric entities	1
2.1.	Contravariant vector	1
2.2.	Covariant vector	2
2.3.	Contravariant tensor of order two	2
2.4.	Covariant tensor of order two	2
2.5.	Mixed tensor of type $(1, 1)$	2
3.	Curves and surfaces in a $V_n$	2
3.1.	Subspaces and hyper surfaces in $V_n$	2
4.	Symmetric and skew-symmetric parts of a tensor $S_{ij}$	3
<b>2</b>	<b>LAPLACE TRANSFORM</b>	<b>5</b>
1	Integral transform	5
2	Laplace transform of some standard functions	5
2.1.	$\mathcal{L}(1)$	5
2.2.	$\mathcal{L}(t)$	6
2.3.	$\mathcal{L}(t^n)$	6
2.4.	$\mathcal{L}(e^{at})$	6
2.5.	Trigonometric functions	6
2.6.	Hyperbolic functions	7
3	Some properties of Laplace transforms	7
3.1.	Linear property	7
3.2.	First shifting (or translation) property	8
3.3.	Second shifting property	8
3.4.	Change of scale property	9
4	Examples based on above properties of Laplace transforms	9
5	Laplace transform of derivative of a function	11
6	Laplace transform of integrals	12
7	Laplace transform of a function $t^n \cdot F(t)$	14
8	Laplace transform of a function $F(t)$ divided by $t$	16
9	Two important theorems	19
10	Laplace transforms of some special functions	20
10.1.	The error function	20

10.2. Laplace transform of <i>sine</i> integral	21
10.3. Laplace transform of <i>cosine</i> integral	22
10.4. Laplace transform of exponential integral	23
10.5. Laplace transform of Bessel's function	23
10.6. Some deductions from $\mathcal{L} J_0(t)$	24
10.7. Unit step function	26
11 Problem Set	26
12 Important formulae	28
<b>3 (INVERSE) LAPLACE TRANSFORM</b>	<b>31</b>
1 Introduction	31
2 Properties of inverse Laplace transform	32
3 Methods to find inverse Laplace transforms	35
3.1. Partial fractions method	35
3.2. Method of differentiation w.r.t. some parameter	36
4 Inverse Laplace transforms of derivatives and integrals	37
5 Convolution theorem	40
6 Problem set	44
<b>4 (APPLICATIONS OF) LAPLACE TRANSFORMS TO DIFFERENTIAL EQUATIONS</b>	<b>53</b>
1 Introduction	53
2 Solution of ODEs with constant coefficients by Laplace transform methods	53
3 Solution of ODEs with variable coefficients	58
4 Problem set	60
<b>5 LINEAR ALGEBRA</b>	<b>71</b>
1 Introduction	71
2 Characteristic equation and eigen values of a matrix	71
3 Minimal polynomial	74
4 Bilinear quadratic and Hermitian forms	75
4.1. Bilinear form on a vector space	75
4.2. Hermitian form on a vector space	76
<b>6 LINEAR PROGRAMMING</b>	<b>77</b>
1 Linear programming	77
2 Graphical method	80

3	Some exceptional cases	85
4	General linear programming problem	87
	4.1. Canonical and standard forms	88
5	Simplex method	89
6	Working procedure of the simplex method	94
7	Problem set	98
<b>7</b>	<b>MATRIX THEORY</b>	<b>101</b>
1	Matrix	101
2	Some special matrices	101
	2.1. Null matrix	101
	2.2. Diagonal matrix	102
	2.3. Scalar matrix	102
	2.4. Identity matrix	102
	2.5. Triangular	103
	2.6. Equal matrix	103
	2.7. Transpose matrix	103
	2.8. Symmetric matrix	103
	2.9. Skew symmetric matrix	104
	2.10. Singular Matrix	104
	2.11. Non-singular matrix	104
	2.12. Row matrix	104
	2.13. Column matrix	104
	2.14. Nilpotent	105
	2.15. Idempotent	105
	2.16. Orthogonal	106
	2.17. Similar	106
	2.18. Conjugate matrix	106
	2.19. Hermitian	106
	2.20. Skew-Hermitian	106
	2.21. Unitary	106
	2.22. Echelon form	106
	2.23. Jordan canonical form	107
3	Addition of two (or more) matrices	107
4	Multiplication of a matrix by a scalar	108
5	Multiplication of matrices	108
6	Elementary operations on matrices	110
	6.1. Comparative features	112
7	Rank of matrix	112
	7.1. Rank of some special matrices	112
8	Inverse of a non-singular matrix	113

8.1. Orthogonal matrix	116
9 Linear equations. Cramer's rule	116
9.1. Two simultaneous linear equations	116
9.2. Three simultaneous linear equations	117
9.3. $n$ simultaneous linear equations	118
9.4. Solution of Eqs. (9.9a) by matrix methods	118
10 Homogeneous linear equations	121
11 Similar matrices and diagonalization of a matrix	122
<b>8 METRIC SPACES</b>	<b>125</b>
1 Definition and examples	125
2 Open spheres in a metric space	131
3 Open sets	134
4 Neighbourhood of a point	137
5 Interior of a set	140
6 Closed sets	142
7 Limit point of a set	145
8 Closure of a set	146
9 Boundary of a set	149
10 Dense and perfect sets	153
11 Continuous maps	156
12 Sequences	161
12.1. Monotone sequences	162
12.2. Convergent, divergent and oscillatory sequences	162
12.3. Cauchy sequence	163
12.4. Complete (metric) space	164
13 Bounded and unbounded sets	165
14 Compact metric space	168
15 First countable (metric) space	169
16 Problem set	170
<b>9 NUMBER SYSTEM</b>	<b>171</b>
1 Natural numbers	171
2 Integers	171
2.1. Odd and even numbers	172
3 Rational Numbers	172
3.1. Approximate values of some irrational numbers	172
4 Prime and composite numbers	172
4.1. Composite number	172

	4.2. Relatively prime numbers	172
5	Algebraic and transcendental numbers	172
6	Complex number	173
7	Algebraic laws for complex numbers	173
8	Modulus and argument of $z$	173
9	Properties of complex numbers	174
10	Logarithm of a complex number	175
<b>10</b>	<b>NUMBER THEORY</b>	<b>177</b>
1	Introduction	177
2	Dawn of arithmetic	177
	2.1. Euclid	178
3	Ancient Greek and the early Hellenistic period	179
	3.1. Eusebius (263 - 339 A.D.)	179
	3.2. Diophantus of Alexandria	180
4	Mathematics in India	181
	4.1. Āryabhaṭa	181
	4.2. Brahmagupta	181
	4.3. Bhāskara I	181
	4.4. Bhāskara II	182
5	Arithmetic in the Islamic golden age	182
6	Western Europe in the middle ages	183
	6.1. Fibonacci	183
	6.2. Early modern number theory.	184
	Pierre de Fermat	
	6.3. Leonhard Euler	185
	6.4. Joseph-Louis Lagrange	186
	6.5. Adrien-Marie Legendre	187
	6.6. Johann Carl Friedrich Gauss	187
7	Sprouting of further subfields	187
	7.1. Elementary tools	188
	7.2. Analytic number theory	189
	7.3. Algebraic number theory	189
	7.4. Diophantine geometry	190
8	Other subfields	192
	8.1. Probabilistic number theory	192
	8.2. Arithmetic Combinatorics	192
	8.3. Computations in number theory	193
9	Applications	194

<b>11 NUMERICAL ANALYSIS</b>	<b>195</b>
1 Numerical solutions of ODEs	195
2 Picard's method for the solution of Eq. (1.1)	195
3 Taylor's series method for the solution of Eq. (1.1)	196
4 Euler's method for the solution of Eq. (1.1)	199
5 Improved Euler's method	200
6 Runge's method for the solution of Eq. (1.1)	201
6.1. Working rule	202
7 Runge-Kutta method	203
7.1. Working rule	203
8 Milne's method	205
9 Adams-Bashforth method	210
<b>12 OPERATION RESEARCH</b>	<b>215</b>
1 Curve fitting	215
2 Method of least squares	215
3 Spline fittings	218
4 Trapezoidal rule for area of a region bounded by a plane curve	220
5 Simpson's rule for an approximate area of a region	221
6 Simpson's <i>three-eighth</i> rule for an approximate area of a region	223
<b>13 POWER SERIES AND EXPANSION OF FUNCTIONS</b>	<b>225</b>
1 Power series	225
1.1. Interval of convergence of a power series	225
2 Maclaurin's series	226
3 Taylor's series	229
<b>14 QUADRATIC FORMS</b>	<b>233</b>
1 A quadratic form	233
<b>15 RIEMANNIAN GEOMETRY</b>	<b>237</b>
1 Introduction	237
2 Metric of a $V_2$	237
3 Non-Euclidean metric	240
4 Riemannian metric	240

5	Associate metric tensor	246
6	Angle between two vectors in $V_n$	248
7	Christoffel symbols	250
8	Covariant differentiation of a contravariant vector	255
9	Covariant differentiation of a covariant vector	257
10	Covariant derivation of tensors	258
11	Commutation formula for a contravariant vector	261
12	Commutation formula for a covariant vector	262
13	Commutation formula for any tensor	263
14	Associate curvature tensor	264
15	Some more properties of curvature tensor and associate curvature tensor	267
16	Contraction of curvature tensor	269
17	Gradient (of a scalar)	273
18	Divergence (of a contravariant vector) 18.1. Divergence of a covariant vector	274 275
19	Curl (of a covariant vector)	277
20	Laplacian of a scalar function	280
21	Geodesics	281
22	One-parameter family of group of transformations 22.1. Transitive and intransitive group of transformations	283 285
23	Infinitesimal transformations	285
24	Infinitesimal motion. Killing equations	286
25	Infinitesimal translation	290
26	Infinitesimal conformal motion (Homothetic)	290
27	Infinitesimal transformations preserving geodesics	291
28	Problem set	292

**16 SEQUENCES AND SERIES**

297

1	Introduction	297
2	Some properties of convergent series	299
3	Root test	301
4	Ratio (or D'Alembert's) test	303
5	Higher ratio (or Raabe's) test	306
6	Logarithmic test	309
7	The series $\sum 1/n^p$ 7.1. Comparison test	311 311
8	Cauchy's condensation test	315
9	Another ratio test	316
10	Infinite products	320

<b>17</b>	<b>SERIES SOLUTIONS OF ODEs</b>	<b>323</b>
1	Introduction	323
2	Ordinary and singular points of an ODE	323
3	Series solutions about singular points	328
	3.1. Frobenius method for a series solution of ODE around a singular point	330
4	Bessel's Equation	332
	4.1. To find a series solution of Eq. (4.1)	333
<b>18</b>	<b>SET THEORY</b>	<b>337</b>
1	Sets	337
2	Some other standard sets	338
3	Symbols and notations	339
	3.1. The symbols $\epsilon$ and $\ni$	339
	3.2. Set inclusion	339
	3.3. Proper subset	339
	3.4. The symbols $\Rightarrow$ and $\Leftrightarrow$	339
	3.5. The symbol $\forall$	340
	3.6. The symbol $\exists$	340
	3.7. The symbol $\cup$	340
	3.8. The symbol $\cap$	340
	3.9. Difference set	341
	3.10. Complement of a set	341
	3.11. Symmetric difference of two sets	342
	3.12. A ring of sets	342
	3.13. The symbol $\circ$	342
4	Algebraic laws	342
	4.1. Binary law (or closure property)	342
	4.2. Associative law	342
	4.3. Identity law	343
	4.4. Inverse law	343
	4.5. Commutative law (or abelian property)	343
	4.6. Cancellation laws	343
	4.7. Distributive laws	343
5	Functions	344
	5.1. Types of maps	345
	5.2. Product of maps	346
	5.3. Identity mapping	346
6	Product of sets	346
7	Partitioning of a set and equivalence relation	348

<i>Contents</i>	xv	
8	Countable and uncountable sets	349
9	Partial order relation	350
10	Total (or linear) order relation	351
<b>19</b>	<b>SOME SPECIAL FUNCTIONS</b>	<b>353</b>
1	Beta function	353
2	Gamma function	355
3	Frullani's integral	364
<b>BIBLIOGRAPHY</b>	<b>367</b>	
<b>INDEX</b>	<b>371</b>	