

CONTENTS

PREFACE		xiii
1	PRELIMINARIES	1
1	Three-dimensional Euclidean space	1
2	Product of vectors	1
3	Derivation of vector-valued functions	5
4	A straight line	6
5	Plane	8
6	Some curved surfaces	8
	6.1. Sphere	9
	6.2. Cone	9
	6.3. Cylinder	10
	6.4. Ellipsoid and hyperboloid	10
	6.5. Paraboloid	11
7	Intersection of two surfaces	12
2	GEOMETRY (DIFFERENTIAL): PART 1 (CURVES IN E_3)	13
1	Parametric representation of a curve	13
2	Surface in E_3	13
3	Linear element of a curve	14
4	Tangent to a curve	14
5	The osculating plane	16
6	Contact of a surface with a curve	19
7	Curvature and the principal normal	22
8	Circle of curvature	25
9	Binormal, torsion and Frenet-Serret formulae	26
10	Formula for torsion of a curve. Plane curve	30
11	Helix. Circular helix	34
12	Locus of center of curvature	39
13	Osculating sphere	41
14	Locus of center of spherical curvature	45
15	Spherical indicatrix of the tangents	48
16	Spherical indicatrix of the binormals	49
17	Involute and evolute	52
	17.1. Involute	52
	17.2. Evolute	53
18	Bertrand curves	56

19	Coordinates in terms of s	59
20	Intrinsic equation of a curve	66
21	Problem set	68
3	GEOMETRY (DIFFERENTIAL): PART 2 (ENVELOPES AND RULED SURFACES)	77
1	Various forms of surfaces	77
	1.1. Tangent plane to a surface	77
2	One-parameter family of surfaces, characteristic curve, envelope and edge of regression	81
3	Ruled surfaces	87
4	Some properties of developables	92
5	Developables associated with space curves	96
6	Problem set	103
4	GEOMETRY (DIFFERENTIAL): PART 3 (CURVATURE OF SURFACES)	119
1	Curvilinear coordinates	119
2	Fundamental magnitudes of first order.	120
	2.1. Christoffel symbols	122
3	Directions on a surface	123
4	Surface normal and fundamental magnitudes of second order	127
	4.1. Surface normal	127
	4.2. The second fundamental form	128
5	Derivatives of $\bar{\mathbf{n}}$	131
6	Curvature of a normal section	134
7	Curvature of an oblique section	136
8	Principal curvatures. Principal directions	137
9	Lines of curvature	144
10	Developables associated with lines of curvature	151
11	Three types of points of a surface and Dupin's indicatrix	154
12	Problem set	157
5	GEOMETRY (DIFFERENTIAL): PART 4 (GAUSS AND MAINARDI – CODAZZI EQUATIONS)	169
1	Gauss formulae	169

2	Gauss characteristic equation	176
3	Mainardi-Codazzi equations	179
4	Problem set	182
6	GEOMETRY (DIFFERENTIAL): PART 5 (SPECIAL CURVES ON A SURFACE)	185
1	Conjugate lines	185
2	Asymptotic lines	189
3	Isometric lines	195
4	Geodesics	198
5	Properties of geodesics. Bonnet's theorem	208
	5.1. Some theorems	208
	5.2. Torsion of a geodesic	210
	5.3. Normal angle	211
6	Geodesic curvature of a curve	216
7	Problem set	223
7	GEOMETRY (FINITE)	229
1	Mathematical logic	229
2	Truth tables	230
	2.1. Unary operations	230
3	Binary operations	231
	3.1. Logical conjunction	231
	3.2. Logical disjunction	232
	3.3. Negation of conjunction $p \wedge q$ and disjunction $p \vee q$	232
	3.4. Negation of conjunction $(\neg p) \wedge (\neg q)$ and disjunction $(\neg p) \vee (\neg q)$	232
4	History of finite geometry	233
	4.1. Axiomatic (or Postulate) System	233
	4.2. Language rules	234
5	Three-point geometry	235
6	Four-point geometry	236
7	Five-point geometry	236
8	Four line geometry	236
9	Axiom set I	237
10	Axiom set II	238
11	Axiom set III and Fano's geometry	239
12	Axioms for the finite geometry of Pappus of Alexandria	241

13	Axioms for the finite geometry of Desargues	242
14	Properties of systems	242
15	Summary of history of finite geometry	244
8	HIOSTORICAL DEVELOPMENT OF EUCLIDEAN GEOMETRY	249
1	Introduction	249
	1.1. Euclid's postulates	249
	1.2. Euclid's axioms	250
	1.3. Elementa	250
	1.4. Empirical and synthetic approaches	251
2	Hilbert's axioms	251
	2.1. Postulates of connection	251
	2.2. Postulates of order	251
	2.3. Postulates of congruence	252
	2.4. Postulate of parallels	253
	2.5. Postulates of continuity	253
	2.6. Dedekind's postulate	253
3	Emergence of Non-Euclidean geometry	254
	3.1. Hyperbolic geometry	254
	3.2. Elliptic geometry	255
9	GEOMETRY (PLANE)	257
1	Congruence between triangles	257
	1.1. Basic Congruence postulate SAS	257
2	Geometric inequalities	259
3	Similarity between triangles	264
10	GEOMETRY (SOLID)	271
1	Lengths	271
2	Areas of plane figures	271
3	Surface areas of solids	272
4	Volume of Solids	273
5	Angle in a regular polygon	273
6	Area and volume of cone and its frustum	274
	6.1. Frustum of a cone	274
7	Pyramid	275
8	Frustum of a Pyramid	277
9	Sphere and its section	278

11	GEOMETRY (SPHERICAL)	281
1	Introduction	281
	1.1. Geometry of Earth's surface	281
	1.2. Equator and meridians	281
	1.3. Longitudes and latitudes	282
12	GEOMETRY (TRANSFORMATION)	283
1	Translation	283
	1.1. Translation in the coordinate plane	283
	1.2. Representation of a translation in matrix form	283
2	Reflection	284
3	Rotation	286
	3.1. Centre of rotation	286
	3.2. Rotation in the coordinate plane	287
4	Dilation	288
	4.1. Composition of a point reflection and a dilation	288
5	Homogeneous coordinates	289
6	Linear transformations in a plane	289
7	Transformations in space. Non-linear transformations	290
8	Rotation of axes in space	292
9	Scalar product of two vectors under a rotation	295
10	Rigid motions	296
11	Affine transformations	300
12	Centro-affine transformations in E_3	302
13	Locally affine transformations	303
14	Isometries	303
	14.1. Transformations preserving congruence	303
15	Direct and opposite isometries	304
16	Groups of transformations	305
	16.1. Product transformations	305
17	Identity transformations	309
18	Inverse transformations	309
19	Transformation group	310
20	A projective space of dimension n	312
21	Projective transformations	314
22	Singular and non-singular projective transformations	315
23	Collineations	315

13	IMPROPER INTEGRALS	319
1	Practical comparison test	319
2	Limit of the integral $\int_a^b f(x) dx$	325
3	Improper integrals of second kind	326
4	Convergence of improper integrals of products of two functions	333
14	EVALAUTION OF LIMITS	339
1	The integral $\int_{-\infty}^{\infty} f(x) . dx$	339
2	The integral $\int_{-\infty}^{\infty} \{ f(x) / g(x) \} . dx$	339
3	The integral $\int_0^{\infty} \{ x^{2m} / (1 + x^{2n}) \} dx$	341
15	UNIFORM CONVERGENCE OF IMPROPER INTEGRALS	349
1	Uniform convergence of improper integrals	349
	1.1. Tests for uniform convergence	349
	1.2. Properties of uniform convergent improper integrals	349
16	JACOBIANS	359
1	Introduction	359
	1.1. General form of Jacobian	360
2	Properties of Jacobians	361
3	Special forms of functions	365
	3.1. First form	365
	3.2. Second form	365
	BIBLIOGRAPHY	367
	INDEX	371