

CONTENTS

PREFACE	xvii
1 PRELIMINARIES	1
1 Results referred in the text	1
2 ALGEBRA (CLASSICAL)	5
1 Law of indices	5
2 Factorization	5
3 Equal fractions	5
4 Progressions	5
4.1 Arithmetic progression	5
4.2 Geometric progression	6
4.3 Harmonic progression	6
5 First n natural numbers	7
6 Factorials	7
7 Permutations and combinations	8
8 Binomial theorem	8
9 Exponential function	9
10 Logarithms	10
10.1 Common logarithm	10
10.2 Some rules for logarithms	10
10.3 Change of base	10
10.4 Expansion of logarithmic functions	11
10.5 Log of some numbers	11
3 ALGEBRAIC STRUCTURES	13
1 Group	13
1.1 Monoid	13
1.2 Groupoid (or semigroup)	13
1.3 Semigroup with identity	13
1.4 Commutative semigroup	13
1.5 Group	13
1.6 Commutative (or abelian) group	13
2 Some properties of groups	14
3 Some special groups	16
3.1 Cyclic group	16
3.2 Transformation group	16

4	Homomorphism and isomorphism of groups	17
5	Coset decomposition of a group and factor group	18
6	Rings, Fields and Integral domains	18
7	Some properties of rings	19
8	Some special rings	20
	8.1 Commutative ring	20
	8.2 Ring with identity	20
	8.3 Division ring	20
	8.4 Field	21
	8.5 Integral domain	21
	8.6 Subring	22
9	Ideals of a ring	22
10	<i>Coset</i> decomposition of a ring and quotient ring	23
11	Homomorphism and isomorphism	24
4	ARITHMETIC	27
	1 Average	27
	2 Percentage	27
	3 Profit and loss	27
	4 Ratio and proportion	27
	5 Inequalities	27
	6 Equal fractions	28
	7 Simple interest	28
	8 Compound interest	28
	9 Work problems	28
	10 Time and watch	28
	11 Fundamental theorem of arithmetic	28
5	CALCULUS (DIFFERENTIAL)	29
	1 Limit of a function	29
	2 Properties of limits	30
	3 Some important limits	31
	3.1 Properties of $f(x) = (1 + 1/x)^x$	32
	4 Continuity of a function	32
	5 Some properties of continuous functions	34
	6 One-sided continuity	37
	7 Differentiation of functions	40
	8 Continuity of a differentiable function	42
	9 Derivatives of some standard functions	44
	9.1 Algebraic functions	44

9.2	Exponential and logarithmic functions	45
9.3	Trigonometric functions	46
9.4	Hyperbolic functions	47
10	Chain rule (<i>methods of substitution</i>)	47
10.1	Logarithmic differentiation	47
11	Derivation of inverse trigonometric functions	48
12	Implicit functions and their derivation	49
13	Successive differentiation	49
14	Leibnitz theorem	51
15	Maxima/minima of a function on a closed interval	53
15.1	Extrema at the end points of interval	53
15.2	Local maxima / minima of a function	54
16	Derivative of $f(x)$ at local extremum points	55
17	Monotonic functions	61
18	Points of local extrema	62
19	Second derivative test for local extrema	63
20	Application of differentiation to calculate rate of change	66
21	Partial differentiation of scalar functions	66
22	Total derivative of a function	69
23	Euler's theorem on homogeneous functions	70
24	Envelope of family of curves	73
24.1	One-parameter family of curves	73
24.2	Envelope of family of curves involving two parameters	74
24.3	General method for more than one parameter	75
24.4	Envelope in polar coordinates	76
25	Involutes and evolutes	77
26	Series and expansion of functions	78
26.1	Power series	78
26.2	Interval of convergence of a power series	79
27	Maclaurin's series	80
28	Taylor's series	82
6	CALCULUS (INTEGRAL)	87
1	Introduction	87
2	Indefinite integrals of some standard functions	87
3	Different methods of integration	90
3.1	Substitution method	90
3.2	$\int \{ 1 / (ax^2 + bx + c) \} dx$	90
3.3	$\int \{(px + q) / (ax^2 + bx + c)\} dx$	91

3.4	Some reduction formulae	92
4	Definite integrals	92
5	Some theorems on definite integrals	94
6	Area of a region by single integration	97
7	Displacement	100
8	Work done by a force	100
9	Volume of solids of revolution	102
10	Area of a region by double integration	103
10.1	In rectangular Cartesian coordinates	103
10.2	In polar coordinates	103
11	Change of order of integration	104
12	Changing order of integration when limits are defined by an inequality	107
13	Triple integration	109
14	Transformation of multiple integrals	111
15	Transformation into polar coordinates	114
15.1	Double integrals	114
15.2	Triple integrals	114
16	Dirichlet's and Liouville's integrals	115
16.1	Particular cases	116
7	COMPLEX VARIABLE	119
1	Functions of a complex variable	119
2	Limit of $f(z)$	119
3	Derivative of $f(z)$ with respect to z	120
4	Analytic functions	120
5	Applications of C-R conditions to Laplace equation	122
6	Integral of a complex function along a curve	124
7	Cauchy's integral theorem	126
8	Cauchy's integral formula	126
9	Integration of a power series and Laurent's series	128
9.1	Series of complex terms	128
10	Singular point of an analytic function. Residue	131
11	Evaluation of real definite integrals by contour integration	133
11.1	Integration around a unit circle	133
11.2	Integration around a small semi-circle	134
11.3	Integration around rectangular contours	135
11.4	Indenting the contours having poles on the real axis	137
12	Conformal transformation and geometrical	139

	interpretation of $w = f(z)$	
13	Some standard conformal transformations	141
	13.1 Translation	141
	13.2 Rotation	141
	13.3 Rotation	142
14	Bilinear transformation	144
	14.1 Invariant points of a bilinear transformation	144
15	Some special conformal transformations	146
	15.1 Transformation $w = z^2$	146
	15.2 The transformation $w = z^n$	147
	15.3 The transformation $w = e^z$	148
	15.4 Transformation $w = \cosh z$	148
	15.5 Joukowski's transformation $w = z + 1/z$	149
16	Complex potential and application of complex analysis to flow problems	151
		157
8	DETERMINANTS	
		157
	1 Determinant	158
	2 Some properties of determinants	
9	DIFFERENTIAL EQUATIONS (ORDINARY)	161
	1 Introduction	161
	2 ODEs of 1 st order and 1 st degree (simple cases)	162
	3 First order and first degree linear differential equations (general cases)	163
	3.1 Separable variables form	163
	3.2 Homogeneous form	163
	3.3 Reducible to homogeneous form	164
	3.4 A linear form	164
	3.5 Reducible to a linear form	165
	3.6 Exact form	166
	3.7 Change of variables	167
	4 Differential equations of the 1 st order but of any degree	167
	4.1 Equations solvable for p	167
	4.2 Equations solvable for y	168
	4.3 Equations solvable for x	168
	4.4 Clairaut's form	169
	4.5 A more general form	169
	5 Linear differential equations of any order with	170

constant coefficients	
5.1 All real and distinct roots	170
5.2 Some equal roots	170
5.3 Complex roots	172
5.4 Pair of roots $\alpha \pm \sqrt{\beta}$	172
6 The particular integral	173
7 Homogeneous linear differential equations of any order	175
7.1 Complementary function	176
7.2 Particular integral	177
8 Differential equations reducible to homogeneous linear form	177
9 Simultaneous differential equations	178
9.1 Simultaneous linear differential equations with constant coefficients	178
9.2 Simultaneous differential eqs. in 3 variables	179
10 Solution of ODE by variation of parameters method	180
10.1 Solution of a linear ODE of I order	180
10.2 Solution of linear ODE of II order with constant coefficients	181
11 Normal form of an ODE	183
10 DIFFERENTIAL EQUATIONS (PARTIAL)	185
1 Introduction	185
1.1 First order PDEs	185
1.2 First order PDEs of general form	188
2 Classification of first order PDEs	188
2.1 Solution of quasi-linear / linear equations	189
2.2 Particular cases of linear ODEs	191
2.3 (Quasi-) linear PDEs involving n independent variables	193
3 Non-linear PDEs of first order	194
4 Second order PDEs with variable coefficients	197
4.1 First type	198
4.2 Second type	199
4.3 Third type	201
5 Second order linear PDEs in mechanics	204
6 Homogeneous linear PDE	207
6.1 Separation of variables	207
7 One-dimensional wave equation	210

7.1	Particular case	214
8	Second order PDEs in n independent variables	215
8.1	Classification of linear PDEs of second order	216
11	DIFFUSION EQUATION	217
1	The diffusion equation	217
2	Heat equation	217
12	DYNAMICS	221
1	Motion of a particle in a straight line	221
2	Uniform acceleration	221
3	Vertical motion under gravity	224
3.1	Downward motion	224
3.2	Upward motion	225
4	Kinetics in two dimensions	226
5	Radial and transverse accelerations	227
6	Tangential and normal components of velocity and accelerations	228
7	Tangential and normal components of velocity and accelerations	229
7.1	Above results in terms of vectors	229
7.2	Circular motion with uniform angular velocity	230
8	Simple harmonic motion	234
8.1	Distance in terms of time	235
8.2	Motion in a vertical circle	236
9	Projectile on a horizontal plane	237
10	Projectile on an inclined plane	240
10.1	Maximum range on inclined plane	241
10.2	Particle projected down an inclined plane	242
11	Energy	244
11.1	Kinetic energy	244
11.2	Potential energy	245
12	Momentum of a particle	246
13	Conservation of linear momentum	247
14	Impact with a fixed surface	248
14.1	Perpendicular impact	248
14.2	Oblique impact	249
15	Direct impact of two spheres	251

16	Relative motion	252
13	FOURIER TRANSFORMS OF FUNCTIONS	255
1	Introduction	255
2	Properties of the Fourier transform	255
3	Fourier series	256
4	Evaluation of Fourier coefficients	257
5	Fourier series in any interval	260
5.1	Interval $[-l, l]$	260
5.2	Interval $[0, \pi]$	261
5.3	A sine series in the interval $[0, \pi]$	263
5.4	Interval $[0, l]$	263
5.5	The interval $[0, 2\pi]$	264
5.6	The interval $[0, 1]$	264
5.7	The interval $[a, b]$	265
6	Fourier series for piecewise functions	266
14	GEOMETRY (COORDINATE: 2-DIMENSIONAL)	269
1	Rectangular Cartesian coordinates	269
2	Distance between two points	269
3	Point dividing a line in a given ratio	270
4	Equation of a straight line	270
4.1	Slope-intercept form	270
4.2	Intercepts on axes	270
4.3	Perpendicular form	271
4.4	Line through a given point	272
4.5	Line through two points	272
5	Angle between two lines	273
6	Perpendicular from a point to a line	274
7	Intersection of two lines	274
8	Circle	275
9	Parabola	275
10	Ellipse	276
11	Hyperbola	277
12	Polar coordinates	277
13	General equation of second degree in x and y	278
15	GEOMETRY (COORDINATE: 3-DIMENSIONAL)	285
1	Rectangular Cartesian coordinates (revisited)	285

1.1	Two-dimensional	285
1.2	Three-dimensional	285
2	Slope and direction cosines of a line	286
3	Line through a given point	286
4	Line through two points	287
5	The coordinate axes and the coordinate planes	288
6	A linear equation	289
6.1	In 2 variables	289
6.2	In 3 variables	290
7	Angle between two lines / planes	290
7.1	Lines with slope m_1, m_2	290
7.2	Lines with general equations	291
7.3	Lines in 3-dimensional geometry	291
7.4	Normal to a plane	291
7.5	Angle between two planes	292
8	Perpendicular from a point	292
8.1	To a line in E_2	292
8.4	To a plane	293
9	Intersection of lines / planes	294
10	Shortest distance between two lines	294
11	Parametric equations of curves	294
12	Sphere	296
13	More general surfaces in E_3	296
13.1	Ellipsoid and hyperboloid	297
13.2	Central conicoid	297
13.3	Paraboloid	297
13.4	Tangent lines and normal to surface	298
14	General equation of second degree in three variables	298
15	Some standard cases	301
16	Intersection of a line and a conicoid	302
17	Tangent plane	303
18	Polar plane of a point with respect to a conicoid	306
19	Enveloping cone of a conicoid	308
20	Centre of a conicoid	309
21	Reduction of equation (13.1) into standard forms	310

BIBLIOGRAPHY **317**

INDEX **321**