

# Contents

<b>1</b>	<b>The Real Numbers</b>	<b>1</b>
1.1	Introduction . . . . .	1
1.2	The Real Numbers $\mathbb{R}$ . . . . .	2
1.3	Bounded subsets of $\mathbb{R}$ : Supremum and Infimum . . . . .	3
1.4	Mathematical Induction . . . . .	6
<b>2</b>	<b>Sequences</b>	<b>11</b>
2.1	Introduction . . . . .	11
2.2	Arithmetic sequences and series . . . . .	13
2.3	Geometric sequences and series . . . . .	14
2.4	Bounded sequences . . . . .	16
2.5	Monotonic sequences . . . . .	17
2.6	Limit of a sequence . . . . .	19
2.6.1	Formal definition . . . . .	19
2.6.2	Basic Properties . . . . .	21
2.6.3	The algebra of limits . . . . .	22
2.7	Basic tests for convergence . . . . .	24
2.7.1	del'Hospital Rules . . . . .	26
2.7.2	Some Basic Limits . . . . .	27
2.8	The Ratio & the Root Tests . . . . .	28
2.9	Sub-sequences & the Bolzano-Weierstrass Theorem . . . . .	31
2.10	Cauchy Sequences . . . . .	33
2.10.1	$\liminf$ and $\limsup$ of a sequence $a_n$ . . . . .	34
2.11	Limit points & Continuity in terms of Sequences . . . . .	35
<b>3</b>	<b>Infinite Series</b>	<b>39</b>
3.1	Series and convergence . . . . .	39
3.1.1	Caychy Series . . . . .	40
3.2	Basic properties . . . . .	41
3.3	The Integral Test . . . . .	43
3.3.1	Estimating the sum of a series . . . . .	47
3.4	Comparison Tests . . . . .	47
3.4.1	The Comparison Test . . . . .	47
3.4.2	The Limit Comparison Test . . . . .	50
3.5	Alternating Series . . . . .	51
3.5.1	Estimating Sums . . . . .	52
3.6	Absolute Convergence . . . . .	53

3.7	The Ratio Test; d'Alembert . . . . .	54
3.8	The Root Test; Cauchy . . . . .	56
3.9	Rearrangements . . . . .	56
3.10	Various Tests . . . . .	58
3.11	Strategy for Testing Series . . . . .	61
3.12	The Cauchy product of Series . . . . .	62
<b>4</b>	<b>Pointwise and uniform convergence</b>	<b>67</b>
4.1	Pointwise Convergence . . . . .	67
4.2	Uniform Convergence . . . . .	68
<b>5</b>	<b>Power Series</b>	<b>71</b>
5.1	Taylor Polynomials & Series . . . . .	71
5.1.1	Motivation & Taylor polynomials . . . . .	71
5.1.2	Taylor & Maclaurin Series . . . . .	74
5.2	Power Series . . . . .	77
5.2.1	Motivation & Definition . . . . .	77
5.2.2	Domain of a Power Series . . . . .	78
5.2.3	Derivatives and Integrals of Power Series . . . . .	81
5.2.4	Multiplying and Dividing Power Series . . . . .	83
5.3	The Binomial Series . . . . .	84
5.4	Applications . . . . .	85
<b>6</b>	<b>Infinite Products</b>	<b>93</b>
6.1	Convergent Infinite Products . . . . .	94
6.2	Tests for Convergence . . . . .	95