CONTENTS

PREFACE **xiii**

1 ORDINARY DIFFERENTIAL EQUATIONS **1**

1 Introduction 1
2 ODEs of 1st order and 1st degree (simple cases) 2
3 First order and first degree linear differential equations (general cases) 4
3.1. Separable variables form 5
3.2. Homogeneous form 5
3.3. Reducible to homogeneous form 5
3.4. A linear form 6
3.5. Reducible to a linear form 6
3.6. Exact form 7
3.7. Change of variables 8
4 Differential equations of the 1st order but of any degree 13
4.1. Equations solvable for p 13
4.2. Equations solvable for y 13
4.3. Equations solvable for x 14
4.4. Clairaut's form 14
4.5. A more general form 15
5 Linear differential equations of any order with constant coefficients 18
5.1. All real and distinct roots 18
5.2. Some equal roots 19
5.3. Complex roots 20
5.4. Pair of roots $\alpha \pm \sqrt{\beta}$ 20
6 The particular integral 22
7 Homogeneous linear differential equations of any order 32
7.1. Complementary function 33
7.2. Particular integral 33
8 Differential equations reducible to homogeneous linear form 35
9 Simultaneous differential equations 37
9.1. Simultaneous linear differential equations with constant coefficients 38
9.2. Simultaneous differential eqs. in 3 variables 38
10 Solution of ODE by variation of parameters method 44

10.1. Solution of a linear ODE of 1st order 44
10.2. Solution of linear ODE of 2nd order with constant coefficients 45
11 Normal form of an ODE 50
12 Problem set 51

2 SERIES SOLUTIONS OF ODEs 75

1 Introduction 75
2 Ordinary and singular points of an ODE 75
3 Series solutions about singular points 80
3.1. Frobenius method 82
4 Bessel's Equation 84
4.1. To find a series solution 84
5 Problem set 87

3 CONVERGENCE OF IMPROPER INTEGRALS 89

1 Practical comparison test 89
2 Limit of the integral $\int_a^b f(x)\,dx$ 94
3 Improper integrals of second kind 96
4 Convergence of improper integrals of products of two functions 102

4 EVALUATION OF LIMITS 109

1 The integral $\int_{-\infty}^{\infty} f(x).\,dx$ 109
2 The integral $\int_{-\infty}^{\infty} \{ f(x)/g(x) \}.\,dx$ 109
3 The integral $\int_0^{\infty} \{ x^{2m}/(1+x^{2n}) \}.\,dx$ 112

5 UNIFORM CONVERGENCE OF IMPROPER INTEGRALS 121

1 Uniform convergence of improper integrals 121
1.1. Tests for uniform convergence 121
1.2. Properties of uniform convergent improper integrals 121

6 BETA AND GAMMA FUNCTIONS 131

1 Beta function 131
2 Gamma function 133
3 Frullani's integral 142

7 DOUBLE INTEGRATION 145

1 Area of a region 145
1.1. In rectangular Cartesian coordinates 145
1.2. In polar coordinates 145
2 Change of order of integration 146
3 Changing order of integration when limits are defined by an inequality 155

8 TRIPLE INTEGRATION 157

1 Introduction 157
2 Transformation of multiple integrals 160
3 Transformation into polar coordinates 165
3.1. Double integrals 165
3.2. Triple integrals 166
4 Dirichlet's and Liouville's integrals 171
4.1. Particular cases 172

9 VOLUME AND AREA OF SURFACES 177

1 Volume by single integration 177
2 Volume by double integration 177
3 Volume by triple integration 179
4 Volume in polar coordinates 186
5 Area of surfaces 187

10 CONVERGENCE OF INFINITE SERIES 191

1 Introduction 191
2 Some properties of convergent series 193
3 Root test 195
4 Ratio (or D'Alembert's) test 197
5 Higher ratio (or Raabe's) test 199
6 Logarithmic test 203
7 The series $\sum 1/n^p$ 204
7.1. Comparison test 205
8 Cauchy's condensation test 208

8.1. Another ratio test 210
9 Infinite products 213
10 Problem set 215

11 FOURIER SERIES 231

1 Fourier series 231
2 Evaluation of Fourier coefficients 231
3 Fourier series in any interval 238
3.1. Interval $[-l, l]$ 238
3.2. Interval $[0, \pi]$ 239
3.3. A sine series in the interval $[0, \pi]$ 243
3.4. Interval $[0, l]$ 244
3.5. The Interval $[0, 2\pi]$ 245
3.6. The interval [0, 1] 246
3.7. The interval $[a, b]$ 247
4 Fourier series for piecewise functions 248

12 FUNCTIONS OF A COMPLEX VARIABLE & THEIR DERIVATION 257

1 Functions of a complex variable 257
2 Limit of $f(z)$ 257
3 Derivative of $f(z)$ with respect to z 258
4 Analytic functions 260
5 Applications of C-R conditions to Laplace's eq. 261

13 COMPLEX INTEGRATION 265

1 Integral of a complex function along a curve 265
2 Cauchy's integral theorem 267
3 Cauchy's integral formula 268
4 Integration of a power series and Laurent's series 270
4.1. Interval of convergence of a power series 270
4.2. Series of complex terms 270
5 Singular point of an analytic function/residue 275
6 Evaluation of real definite integrals by contour integration 280
6.1. Integration around a unit circle 280
6.2. Integration around a small semi-circle 282
6.3. Integration around rectangular contours 283
6.4. Indenting the contours having poles on the real axis 285

14 CONFORMAL TRANSFORMATION 289

1 Geometrical interpretation of $w = f(z)$ 289
2 Some standard conformal transformations 292
2.1. Translation 292
2.2. Rotation 293
2.3. Inversion and Reflection 293
3 Bilinear transformation 296
3.1. Invariant points of a bilinear transformation 297
4 Some special conformal transformations 300
4.1. Transformation $w = z^2$ 300
4.2. The transformation $w = z^n$ 302
4.3. The transformation $w = e^z$ 302
4.4. Transformation $w = \cosh z$ 303
4.5. Joukowski's transformation $w = z + 1/z$ 304

15 COMPLEX POTENTIAL 309

1 Application of complex analysis to flow problems 309

BIBLIOGRAPHY 315

INDEX 317